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INTEGRO-DIFFERENTIAL CHARGES AND CURRENTS DISTRIBUTION ON THE FRACTAL MEDIUM TOPOLOGY

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ABSTRACT

We will consider the geometric and physical aspects that permit introduction of the so-called α -characteristics for studying the behavior of electromagnetic field components in the vicinity of a set of points with fractal properties. To estimate the α -characteristics, possible algorithms are formulated, namely a geometric one involving evaluation of the Hausdorff measure and an analytical algorithm permitting the Hausdorff measure to be evaluated through application of fractional derivatives and integrals.

INTRODUCTION

A great majority of real physical systems are of fractal nature in the respective range of scale sizes, characterized by the values of one or more respective fractal dimensions. Studies of a number of physical phenomena (such as small-angle scattering of X-rays, anomalies in the power-law dependences upon frequency of the electrical resistance or electrical energy dissipation [1], energy radiation and interaction of electromagnetic waves with impedance surfaces [2, 3], etc) have revealed the close relation of their performance to fractal properties of the boundaries and media involved. The possibility of the fractional calculus application to electrostatics was demonstrated in paper [4].

FORMULATION

The well known technique of approximating to non-coordinate boundaries through covering the surface with simple compacts (like rectangles, circles, or ellipses) [5] permits application of numerical algorithms for solving boundary-value problems of electrodynamics.

Let us extend the technique of covering the boundaries and domains of existence of the electromagnetic field to the case of a smooth contour possessing fractally distributed geometric points over its certain section (physically, a highly jagged (rough) portion of the contour). To that end, we will consider a model of the contour section showing the properties of local uniformity and local self-similarity. Let fractal portion of the contour be approximated to with a segmented line with the links $\Delta x_{i(k)}$ of constant length and the ends lying on the contour (k -number of covering generation). To represent the fractal contour approximately with points of the segmented line, let us cover it with a segmented line with links of a smaller length, $\Delta x_{i(k+1)} < \Delta x_{i(k)}$. Apart from higher order small values, the number $N_{\Delta x_{i(k+1)} \Delta x_{i(k)}}$ of the vertices of the segmented line with the link

length $\Delta x_{i(k+1)}$ lying within one link of the segmented line with the link length $\Delta x_{i(k)}$ will be equal to $N_{\Delta x_{i(k+1)}\Delta x_{i(k)}} = f(\Delta x_{i(k)} / \Delta x_{i(k+1)})$. For a covering line with $\Delta x_{i(k+2)} < \Delta x_{i(k+1)}$, we obtain similarly $N_{\Delta x_{i(k+2)}\Delta x_{i(k+1)}} = f(\Delta x_{i(k+1)} / \Delta x_{i(k+2)})$. On the other hand, $N_{\Delta x_{i(k+1)}\Delta x_{i(k)}} \times N_{\Delta x_{i(k+2)}\Delta x_{i(k+1)}} = N_{\Delta x_{i(k+2)}\Delta x_{i(k)}}$. Introducing the notation $u = \Delta x_{i(k)} / \Delta x_{i(k+1)}$ and $v = \Delta x_{i(k)} / \Delta x_{i(k+2)}$ we arrive at a functional equation $f(x)f(y/x) = f(y)$ whose smooth solution is unique in the form of a power law, $f(x) = x^\alpha$. In this connection we are able to use the generalized measure of the manifold magnitude involving choice of a trial power function $h(r) = \gamma(\alpha) \times r^\alpha$, and covering of the multitude of points under study with elements B_i of length r_i , with formation of the Hausdorff α -measure $H^\alpha(E) = \lim_{\varepsilon \rightarrow 0} H_\varepsilon^\alpha(E) = \inf \{ \gamma(\alpha) \sum_i r_i^\alpha : E \subset \bigcup B_i, r_i < \varepsilon \}$. That can serve as a measure of the extent and curvature of the continuous limiting line.

RESULTS

The charge (current) density $j_{\Delta x_{i(k+m)}}$ of $(k+m)$ generation is determined as

$$j_{\Delta x_{i(k+m)}} = j_{\Delta x_{i(k)}} N_{\Delta x_{i(k+m)}\Delta x_{i(k)}}; j_{\Delta x_{i(k+m)}} \frac{\Delta x_{i(k+m)}}{(\Delta x_{i(k+m)})^{1-\alpha}} = j_{\Delta x_{i(k)}} \frac{\Delta x_{i(k)}}{(\Delta x_{i(k)})^{1-\alpha}}.$$

The charge (current) in fractal set is determined now by the Riemann - Liouville fractional integral $J(x) = ({}_a I_x^\alpha j)(x)$.

Differintegral $D^\alpha j$ determines some the differintegral forms of a degree α on Ω with value in \bar{J} (imaging Ω in $L^\alpha(\Omega \subset \bar{E}, \bar{F})$): $\bar{\omega}^\alpha(x) \cdot (\bar{X}) = (D^\alpha j)(x) \cdot \bar{X}$.

Theorem. If $\varphi_1, \varphi_2, \dots, \varphi_p$ - scalar differentiated functions on Ω , the fractional differintegral forms $d^\alpha \varphi_1 \wedge d^\alpha \varphi_2 \wedge \dots \wedge d^\alpha \varphi_p$ concerning some coordinate system in E can be represented as

$$d^\alpha \varphi_1 \wedge d^\alpha \varphi_2 \wedge \dots \wedge d^\alpha \varphi_p = \sum_{1 \leq i_1 < \dots < i_p \leq N} \frac{D^\alpha(\varphi_1, \dots, \varphi_p)}{D^\alpha(x_{i_1}, \dots, x_{i_p})} dx_{i_1} \wedge dx_{i_2} \wedge \dots \wedge dx_{i_p}.$$

If $j^\alpha(x)$ - function in a coordinate neighbourhood (U, x) on E , the α - forms ω^α in this neighbourhood is noted as $j^\alpha(x) dx_1 \wedge \dots \wedge dx_m$.

If the support of $j^\alpha(x)$ belongs to U , on definition: $\int_E j^\alpha = \int_{x(U)} j^\alpha(x) dx_1 \wedge \dots \wedge dx_m$.

The α - volume forms $d^\alpha V = d^\alpha x_1 \wedge \dots \wedge d^\alpha x_m$ on a m -dimensional Riemannian manifold E induces a borel measure, which coincides with the Hausdorff-measure $H^\alpha(U) = \int_U d^\alpha V$ for any open set $U \subset E$. Hence, for any integrable α - forms j^α on

E equality $\int_E j^\alpha = \int_E \langle j^\alpha, \tau_E \rangle dH^\alpha$, where τ_E - vector, defining tangential plane is valid.

We have installed the formula of connection between integrals of the second type (from the α - forms) and integrals of the first type (in respect of Hausdorff-measure).

The further development of the theory is carried out on the basis of interpretation of Dirac delta-function, which is determined as the α - forms.

Electromagnetic field in fractal medium follows Maxwell (Abel) equations in the terms of α - forms and α -characteristics:

$$d^\alpha \vec{E}^{(\alpha)} = -\frac{\partial}{\partial t} \vec{B}^{(\alpha)} - \vec{j}_m^{(\alpha)}, \quad d^\alpha \vec{H}^{(\alpha)} = \frac{\partial}{\partial t} \vec{D}^{(\alpha)} + \vec{j}_e^{(\alpha)}, \quad d^\alpha \vec{D}^{(\alpha)} = \rho_e^{(\alpha)}, \quad d^\alpha \vec{B}^{(\alpha)} = \rho_m^{(\alpha)}.$$

We obtain fractional Green's function for the Helmholtz equation in the terms of the α - characteristics with the relevant fractal boundary conditions [6-8].

CONCLUSION

Generalizing the schemes outlined to include α -differintegral forms Dirac delta-function will promote construction and further analysis of such mathematical models that would permit an adequate description of actual electromagnetic processes at fractal boundaries or in fractal media themselves (e.g., in the problems concerning «artificial» dielectrics, complex media and metamaterials, or power emission by «thick» contours and surfaces, etc).

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